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ABSTRACT

Described is a study concerned with the mode of presentation of printed mathematical word problems. Tenth grade students were given twenty word problems to solve, presented in one of three ways: (1) prose only, (2) prose with an accurate picture included, or (3) prose with a distorted picture. Experimental results showed that the group with an accurate picture performed significantly higher (.005) than the group with no picture, which in turn performed significantly higher (.005) than the group with a distorted picture. The subjects' intelligence scores, reading scores, and grade averages in the preceding year's mathematics courses were related to the subject's experimental achievement. (RS)

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THE EFFECTS OF DIFFERING PRESENTATIONS OF MATHEMATICAL WORD PROBLEMS UPON THE ACHIEVEMENT OF TENTH GRADE STUDENTS

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Kilpatrick (1969) indicated that increasing the ability of students to solve problems has long been a goal of mathematics instruction. The conferees of the Cambridge Conference on School Mathematics stated that the solving of mathematics problems was going to continue to be important in the mathematics curriculum. Regarding the preparation of textbooks for mathematics classes, the conferees stated (Goals for School Mathematics, 1963)

> ...problem materials should be considered at least as important as the text proper; and it should get at least half of the time and attention of the authors.

This article is a report of a research study concerned with the mode of presentation of printed mathematical word problems. The study's specific concern is the

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effect of presenting a pictorial representation of the problem situation.

Riedesel (1969) included the following in his list of aids in the improvement of problem solving:

> Make use of drawings and diagrams as a technique to help pupils solve problems. This technique is helpful to pupils of all ability levels. It forces the child to consider the problem situation.

Other educators have also stressed the importance of using pictorial aids in problem solving (Henderson and Pingry, 1953; Bloom and Broder, 1950; Young, 1968; Trueblood, 1969).

With respect to the use of a pictorial representation, Bloom and Broder (1950) suggested the following:

1. Don't let the physical format confuse the students.
2. If you use diagrams use them correctly.
3. Arrangement of the data can distract from the meaning of the problem.
4. Indicate clearly how the questions are to be answered.

Young (1968), against the purposeful introduction of error into a problem solving situation, made the following statement:

> Neat, orderly work is always helpful, and figures drawn at least fairly accurately are often suggestive.
>
> It is well not to try to correct the consequences of an error discovered, but work afresh.

Brownell (1951), in his general discussion of problem solving, suggested the following:

> Part of the real experience in problem solving is the ability to differentiate between the reasonable and the absurd, the logical and the illogical. Instead of being 'protected' from error, the child should many times be exposed to error and be encouraged to detect and demonstrate what is wrong, and why.

Bigge (1964), in agreement with Brownell, wanted mistakes to be used to advantage. Bigge (1964) made the following comment about mistakes:

> Making mistakes often encourages us to re-examine something we had regarded as true. Teachers usually do not let students make enough mistakes.

The preceding discussion indicates the existence of an area of disagreement with respect to the use and accuracy of pictorial representations of problem situations.

Methods

A testing instrument was developed to gather data which would support or reject the contention that subjects' achievement in solving printed mathematical word problems is affected by the presence of a pictorial representation of the problem situation, regardless of the accuracy of the pictorial representation.

Test Development

Forty problems were selected from the Y- and Z-population test batteries of the National Longitudinal Study of Mathematical Abilities (NLSMA) and three mathematical textbooks (Dolciani, Berman, and Freilich, 1962; Jurgensen, Donnelly, and Dolciani, 1965; Vannatta, Goodwin, and Fawcett, 1962) on the basis of the following two criteria:

...the problems could be solved without a pictorial representation of the problem situation

...the problems could be solved even if the pictorial representation of the problem situation were distorted.

A pilot study was conducted to determine if the administrative procedures would work in one fifty-five minute class, to find the twenty problems that discriminated best, and to improve the sets of distractors. The twenty problems that discriminated best were then used to construct three forms of the final testing instrument.

Form A of the final testing instrument contained only the prose description of the problem situations, each with five distractors. Form B contained an accurate pictorial representation for each item, in addition to the same information included in Form A. Form C

contained a distorted pictorial representation for each item, in addition to the same information included in Form A.

For each item there was a set of five distractors. In each set of distractors there was one called the "distorted answer". The distorted answer was the choice that would have been correct had the distorted pictorial representations presented in Form C been correct.

Figure 1 contains an example problem from the final testing instrument.

Insert Figure 1 about here

Administration

The test was administered early in the fall semester of the school year to 322 tenth grade students. The tests were administered in the students' usual mathematics classrooms by the students' regular mathematics teachers. The test booklets were arranged in random order using a table of random numbers and distributed to the students in the students' ordinary seating arrangement.

The teachers were directed to answer questions of a mechanical nature, i. e., questions concerning

the sharpening of the provided pencils, typographical errors that might be found, etc. The subjects were not told that the pictorial representations in Form C were distorted. At the close of the testing period and in the next class period the teachers answered any questions the students had concerning the testing period.

Background Material

Reading scores (California Achievement Test), I. Q. scores (California Mental Maturity Test for Junior High School), and grades in ninth grade mathematics courses were collected for each student in order to test the effect of the variables on achievement.

Results

The results are divided into two main sections: (1) test analysis and (2) test of the hypotheses.

Test Analysis

The means, standard deviations, and reliability

coefficients of the three forms of the testing instrument are presented in Table 1.

Insert Table 1 about here

There were 114 subjects in Group A, the group that was administered Form A; 96 subjects in Group B, the group that was administered Form B; 112 subjects in Group C, the group that was administered Form C. Table 2 contains the data that represent the performances of each group on each item, as well as how many times Group C chose the distorted answer.

Insert Table 2 about here

Due to the differences in the size of each group it is useful to compare the percentage correct for each group on each item. Table 3 contains the means and adjusted means in percentage form for each group on each item. In Table 3 one will also find the number of non-answers for each item.

Insert Table 3 about here

The data in Table 3 show that Group B scored higher on all the items except 4, 12, and 15. Group C had the lowest percentage on all the items except 2, 6, 9, 12, and 18. On twelve of the twenty items Group B's

percentage was at least twice Group C's percentage. Again referring to Table 3, one notices that on only one item, item 12, was Group C's percentage greater than Group B's percentage and the difference was 1.0 percent.

The data in Table 3 also show that the subjects in Group C favored the distorted answer over the correct answer on all items except 4, 5, 6, and 15. On only five items did over 25% of Group C choose the correct answer while for fifteen items over 25% of Group C chose the distorted answer, and for five of those fifteen items over 50% of Group C chose the distorted answer.

Test of the Hypotheses

The study tested two hypotheses:

1. The subjects' achievement in solving printed mathematical word problems is affected by the presence of a pictorial representation of the problem situation, regardless of the accuracy of the pictorial representation.

2. The subjects' achievement in solving printed mathematical word problems is affected by I. Q. scores, reading scores, and grade averages in the preceding

year's mathematics courses, regardless of the method used to present the mathematics word problems.

To test the first hypothesis the means of the three groups were tested by analysis of variance. The analysis can be found in Table 4.

Insert Table 4 about here

The source of the variance, indicated by the significant F-ratio in Table 4, was determined by applying a modification of Duncan's New Multiple Range Test. The modified version is one, developed by Kramer (1956), which is appropriate with unequal cell size. The final ranking showed that Group B's performance was significantly higher than Group A's performance and Group A's performance was significantly higher than Group C's performance. The level of significance was .005.

The second hypothesis was tested using double classification analysis of variance. Table 5 contains the test of the means when the three groups were divided according to I. Q. scores. Figure 2 is a graphical representation of the means of the three groups divided according to I. Q. scores. I. Q. scores were available for 236 of the 322 subjects in the study. The average I. Q. was 108.

Insert Table 5 and Figure 2 about here

Table 6 contains the comparison of the means when the three groups were divided according to reading scores. Figure 3 is a graphical representation of the means of the three groups divided according to reading scores. Reading scores were available for 244 of the 322 subjects. The average reading score was 10.1.

Insert Table 6 and Figure 3 about here

The test of the means when the three groups were divided according to grade averages in the preceding year's mathematics courses is presented in Table 7. Figure 4 is a graphical representation of the means of the three groups divided according to grade average in the preceding year's mathematics courses. A complete set of grades was available for 249 of the 322 subjects.

Insert Table 7 and Figure 4 about here

In all three applications of double classification analysis of variance significant main effects were noted. The data supported both hypotheses.

The data support the conjectures that the presentation of an accurate pictorial representation of

the problem situation facilitates student achievement and the presentation of a distorted pictorial representation of the problem situation debilitates student achievement. The subjects' I. Q. scores, reading scores, and grade averages in the preceding year's mathematics courses were related to the subjects' achievement, regardless of the method used to present the word problems.

In all three applications of double classification analysis of variance significant interaction effects were noted. The ordinal interaction effects demonstrated that the variables had more effect in the high I. Q. group than the other two I. Q. groups. However, the ranking of the groups was not altered. Equivalent results were obtained for the reading and grade average groups.

Discussion

The results discussed in the last section must be interpreted in the light of the fact that the students were in a testing situation. Great care must be taken in transferring the information obtained to the classroom. Since each of the problems could be

solved without a pictorial representation, the data seem to show that the subjects tended to place emphasis on the pictorial representation rather than the prose description of the problem situation.

Since students, at least in a testing situation, tend to assume the information presented is correct, the teachers should emphasize, as some educators have advocated (Brownell, 1951; Bigge, 1964), that separating the logical from the illogical is part of solving problems.

The effect of presenting the distortion in a testing situation was so strong that the high reading group taking Form C scored lower than the low reading group taking either Form A or Form B. An equivalent result was obtained for the grade average variable.

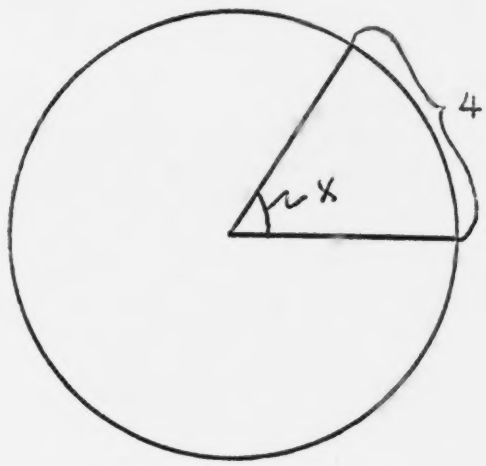
The effect of presenting an accurate pictorial representation as an aid in a testing situation was so strong that the low grade average group taking Form B scored higher than any grade average group taking either of the other two forms.

The circumference (distance around) of a circle is 24 and the length of a minor arc is 4.

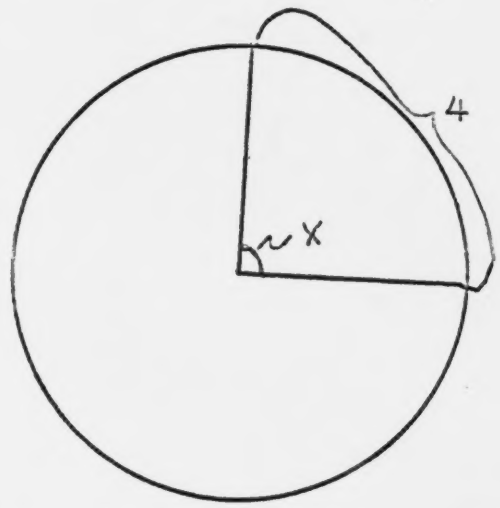
What is the measure (in degrees) of the central angle formed by the two radii which intersect the circle to form the arc?

(A) 24
(B) 30
(C) 45
(D) 60
(E) 90

Form B*



Form C*



*Form A contained only the prose description of the problem situation and the five distractors. Form B contained the same information as Form A, in addition to the pictorial representation on the left. Form C contained the same information as Form A, in addition to the pictorial representation on the right.

Figure 1
Example Problem

TABLE 1

Total Test Means, Standard Deviations, and

Reliability Coefficients

Form	Mean	Standard Deviation	Cronbach's Alpha
A	4.94	2.12	.40
B	8.56	3.31	.70
C	3.79	2.13	.53

TABLE 2

Comparison of the Performances of the

Subjects of All the Groups

Item	Number Choosing Correct Answer			Number Choosing Distorted Answer
	A	B	C	C
1	19	21	9	94
2	22	32	30	52
3	31	85	22	75
4	19	14	13	11
5	65	61	64	6
6	20	25	24	15
7	18	71	4	91
8	19	56	6	61
9	36	51	41	51
10	24	41	3	49
11	32	42	20	58
12	49	31	37	52
13	12	22	8	31
14	35	57	33	39
15	20	13	13	9
16	48	56	27	29
17	16	42	15	30
18	15	47	21	45
19	22	26	11	38
20	21	20	16	17

TABLE 3

Item Analysis: Means, Adjusted Means,

Number Not Answered

Item	Mean Form			Adjusted Mean Form			No Answer Form		
	A	B	C	A	B	C	A	B	C
1	17.3*	22.1*	8.2*	17.3*	22.1*	8.3*	0	0	1
2	20.0	33.7	27.3	20.0	34.0	27.3	0	1	0
3	28.2	89.5	20.0	28.2	89.5	20.0	0	0	0
4	17.3	14.7	11.8	17.4	14.9	11.8	1	1	0
5	59.1	64.2	58.2	59.6	64.9	58.7	1	1	1
6	18.2	26.3	21.8	18.7	26.3	22.0	3	0	1
7	16.4	74.7	3.6	16.4	74.7	3.6	0	0	0
8	17.3	58.9	5.5	17.4	58.9	5.5	1	0	0
9	32.7	53.7	37.3	32.7	53.7	37.3	0	0	0
10	21.8	43.2	2.7	21.8	43.6	2.7	0	1	0
11	29.1	44.2	18.2	29.4	44.2	18.2	1	0	0
12	44.5	32.6	33.6	45.4	32.6	33.6	2	0	0
13	10.9	23.2	7.3	11.1	23.4	7.3	2	1	0
14	31.8	60.0	30.0	32.1	60.0	30.0	1	0	0
15	18.2	13.7	11.8	19.2	13.8	11.9	6	1	1
16	43.6	58.9	24.5	45.7	58.9	24.5	5	0	0
17	14.5	44.2	13.6	15.4	45.2	13.6	6	2	0
18	13.6	49.5	19.1	14.4	50.0	19.1	6	1	0
19	20.0	27.4	10.0	21.4	27.7	10.0	7	1	0
20	19.1	21.1	14.5	20.4	21.7	15.0	7	3	3
					Total Number Not Answered:		49	13	7

*The numbers in these columns are percentages.

TABLE 4

Comparison of the Group Means:

Analysis of Variance Table

Source	Mean Square	df	F-ratio
Total	11.37	321	
Groups	612.64	2	80.62*
Error	7.60	319	

*$p < .005$

TABLE 5

Comparison of the Means When

The Groups are Divided by I. Q. Scores

Source	Mean Square	df	F-ratio
Total	11.52	226	
Between	173.69	8	
Group	502.55	2	90.25*
I. Q.	135.33	2	24.30*
Group X I. Q.	28.43	4	5.11*
Error	5.57	218	

*$p < .005$

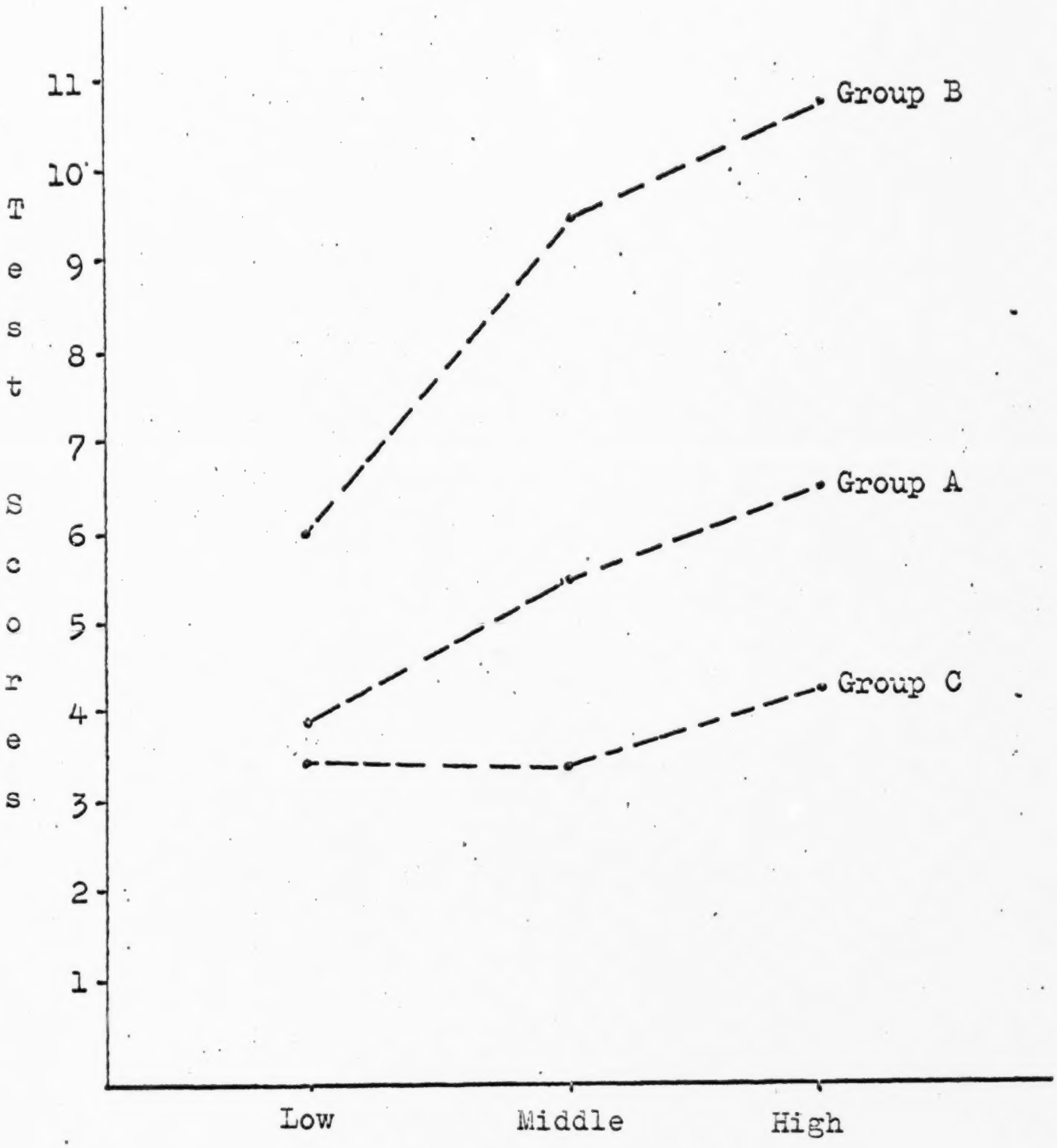


Figure 2
Achievement as a Function of
Group Membership and I. Q.

TABLE 6

Comparison of the Means When the Groups

Are Divided By Reading Scores

Source	Mean Square	df	F-ratio
Total	11.33	236	
Between	165.70	8	
Groups	466.07	2	78.80*
Reading	134.91	2	22.81*
Groups X Reading	30.91	4	5.23*
Error	5.91	228	

*p < .005

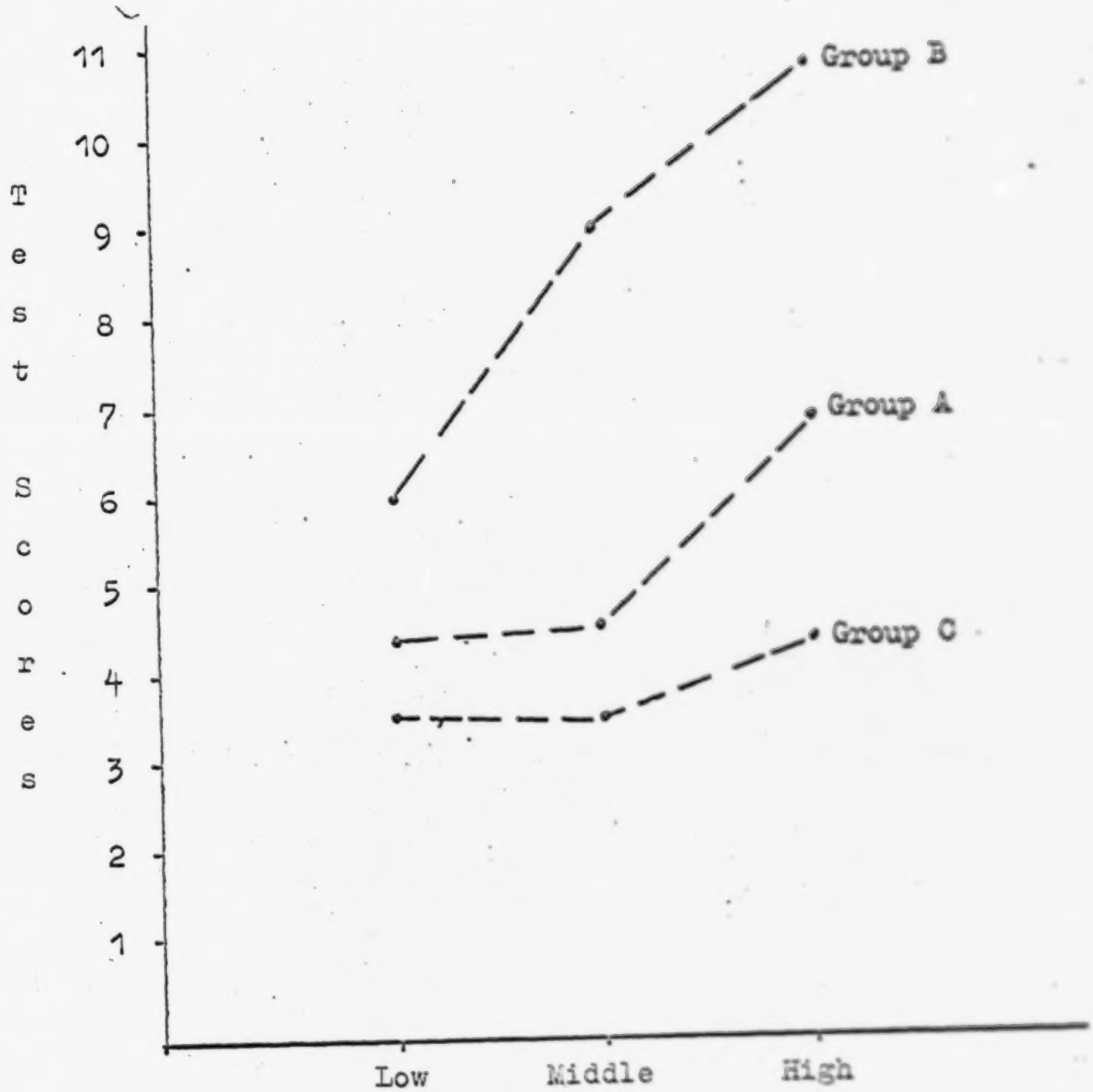


Figure 3
Achievement as a Function of
Group Membership and Reading

TABLE 7

Comparison of the Means When the Groups

Are Divided By Grade Average

Source	Mean Square	df	F-ratio
Total	11.34	241	
Between	146.82	8	
Groups	461.86	2	69.08*
Grades	74.27	2	11.11*
Group X Grades	25.56	4	3.82*
Error	6.69	233	

*p < .005

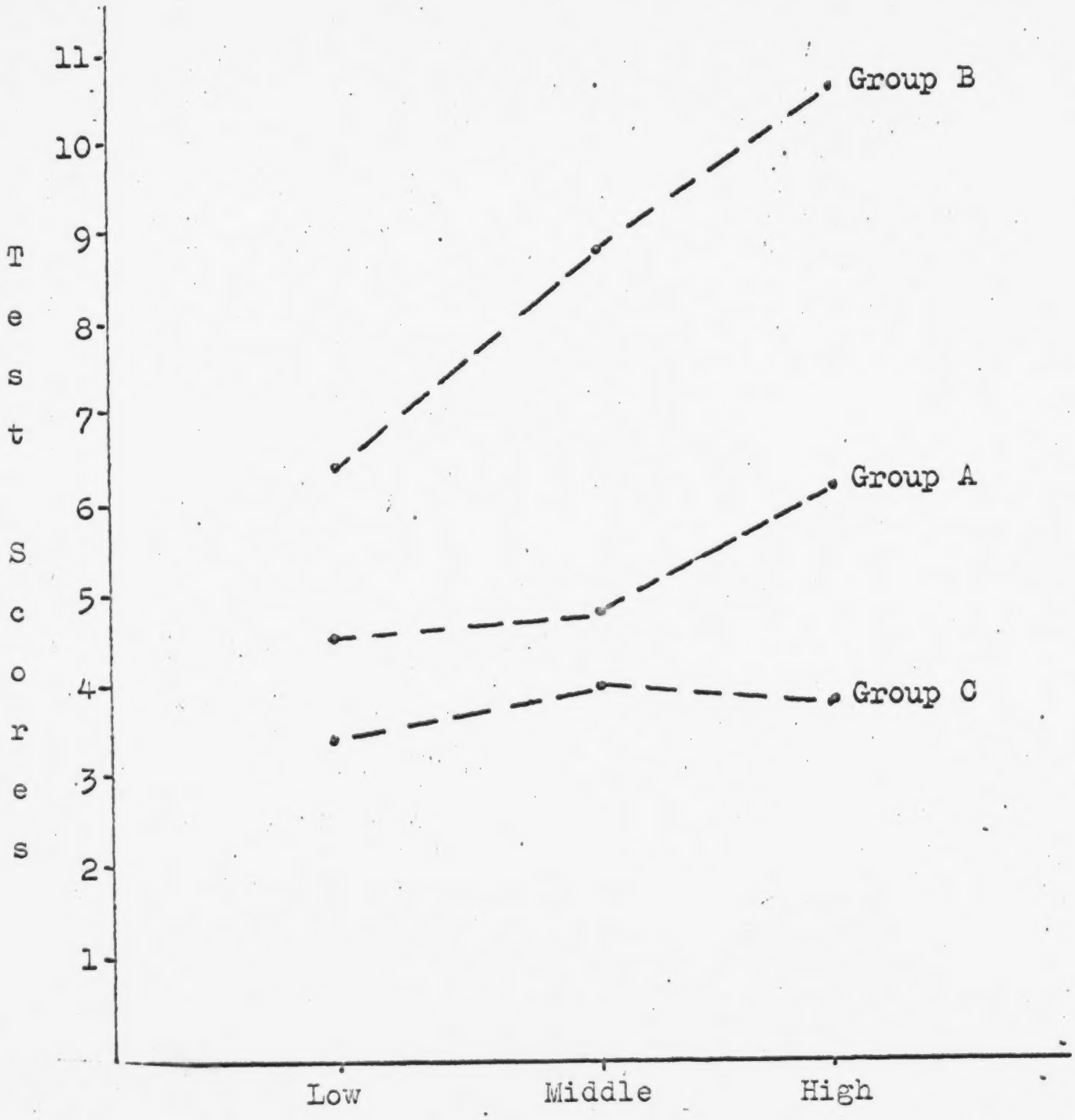


Figure 4
Achievement as a Function of Group Membership and Grade Average

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